Outline

Week 6: Matrix Multiplication and Linear Transformation

Course Notes: 4.1,4.2

Goals: Learn the mechanics of matrix multiplication and linear transformation, and use matrix multiplication to describe linear transformations.

Matrix Anatomy

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}$$

A matrix with 3 rows and 4 columns is a 3 by 4 matrix.

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Here, $a_{3,2} = 6$

Addition and Scalar Multiplication

Addition and scalar multiplication work the way you want them to.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 1 & 5 & -1 \\ 8 & 6 & 6 & 2 \\ 3 & -1 & 2 & -3 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1+2 & 2+1 & 3+5 & 4-1 \\ 2+8 & 4+6 & 6+6 & 8+2 \\ 3+3 & 6-1 & 9+2 & 12-3 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 8 & 3 \\ 10 & 10 & 12 & 10 \\ 6 & 5 & 11 & 9 \end{bmatrix}$$

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$$10A = \begin{bmatrix} 10 & 20 & 30 & 40 \\ 20 & 40 & 60 & 80 \\ 30 & 60 & 90 & 120 \end{bmatrix}$$

Matrix Multiplication

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 11 \\ 10 & 22 \end{bmatrix}$$

In the product, the entry in the ith row and jth column comes from dotting the ith row and jth column of the matrices being multiplied.

$[1, 2, 3] \cdot [1, 2, 0] = 5$

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Another Example

$$\begin{bmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 3 & 0 \\ 1 & 2 \end{bmatrix} =$$

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$$\begin{bmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 3 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 4 & 7 \\ 6 & 5 \end{bmatrix}$$

Wait but... why

$$\begin{bmatrix} 1x_1 + 2x_2 + 3x_3 + 4x_4 \\ 5x_1 + 6x_2 + 7x_3 + 8x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

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$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 5 & 6 & 7 & 8 & 2 \end{array}\right]$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}, \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \qquad b = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$Ax = b$$

Dimensions

$$\begin{bmatrix} * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix} = \begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$

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If A is an m-by-n matrix, and B is an r-by-c matrix, then AB is only defined if $n = r$. If $n = r$, then AB is an m-by-c matrix.

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Can you always multiply a matrix by itself?

Properties of Matrix Multiplication

One important property DOESN'T hold.

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 7 & 5 \\ 3 & 0 \end{bmatrix} =$$

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Suppose the matrix product AB exists. Does the product BA also have to exist?

Properties of Matrix Algebra

The other properties hold as you would like. (Page 128, notes.)

1. $A + B = B + A$
2. $A + (B + C) = (A + B) + C$
3. $s(A + B) = sA + sB$
4. $(s + t)A = sA + tA$
5. $(st)A = s(tA)$
6. $1A = A$
7. $A + \mathbf{0} = A$ (where $\mathbf{0}$ is the matrix of all zeros)
8. $A - A = A + (-1)A = \mathbf{0}$
9. $A(B + C) = AB + AC$
10. $(A + B)C = AC + BC$
11. $A(BC) = (AB)C$
12. $s(AB) = (sA)B = A(sB)$

Examples

Simplify the following expressions.

1) $$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\1 & 2 & 3\end{bmatrix}\begin{bmatrix}8 & 9 & 8\\9 & 8 & 9\\8 & 9 & 8\end{bmatrix} + \begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\1 & 2 & 3\end{bmatrix}\begin{bmatrix}-8 & -9 & -8\\-9 & -8 & -9\\-8 & -9 & -8\end{bmatrix}$$

2) $$\left(\begin{bmatrix}33 & 44\\55 & 66\end{bmatrix}\begin{bmatrix}5 & 1\\7 & 0\end{bmatrix}\right)\begin{bmatrix}0 & 0\\1 & 1\end{bmatrix}$$

3) $$2.8\begin{bmatrix}15 & 0 & 38\\9 & 10 & 11\\8 & 7 & 6\end{bmatrix} + 5.6\begin{bmatrix}-2.5 & 0 & 1\\0.5 & 0 & -0.5\\1 & 1.5 & 2\end{bmatrix}$$

More on Dimensions

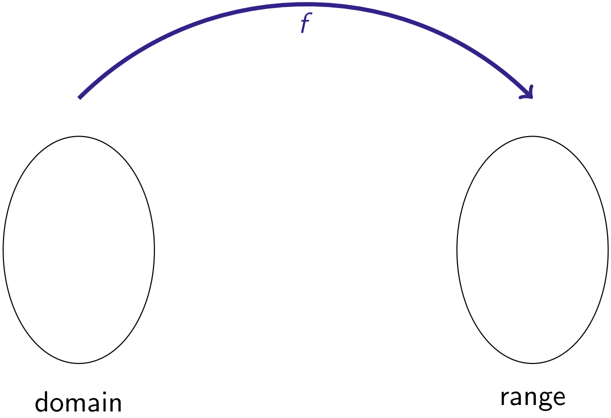
Suppose A is an m-by-n matrix, and B is an r-by-c matrix.

If we want to multiply A and B,
what has to be true about m, n, r, and c?

If we want to add A and B,
what has to be true about m, n, r, and c?

If we want to compute $(A + B)A$,
what has to be true about m, n, r, and c?

Functions and Transformations



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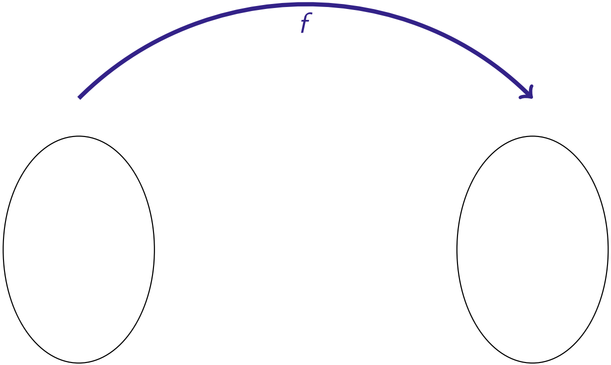
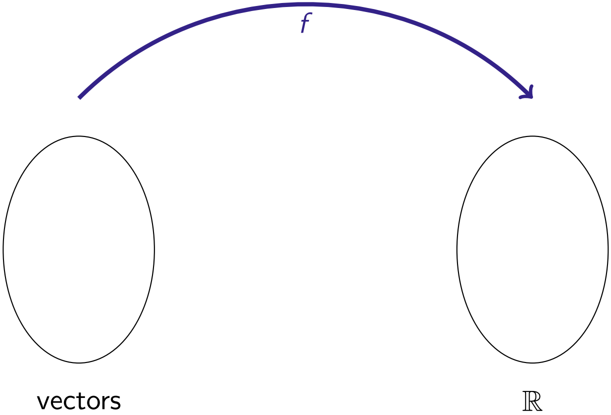


$$f(v) = \|v\|$$

Functions and Transformations



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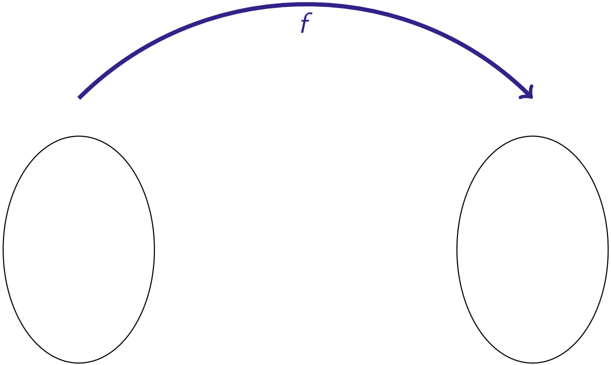
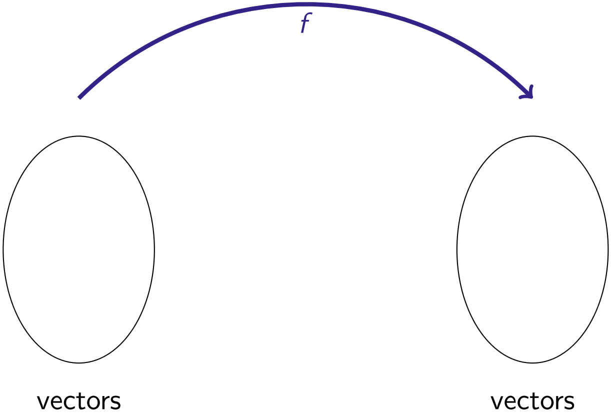


$$f(v) = 3v$$

Functions and Transformations



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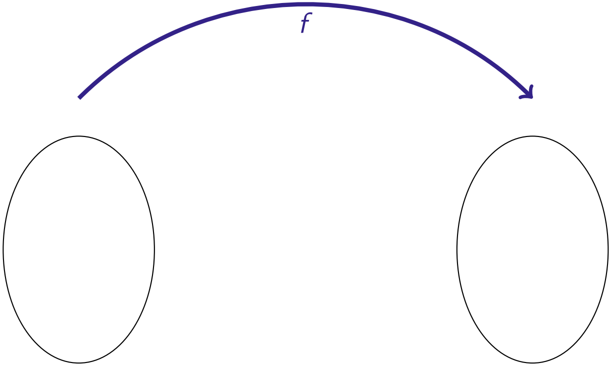

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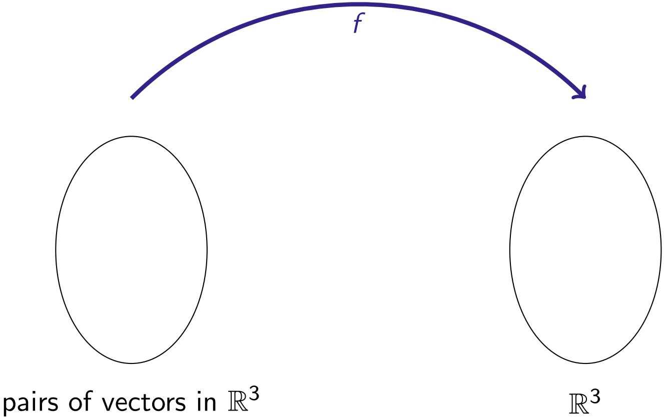

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Linear Transformations

Definition

A transformation T is called **linear** if, for any $\mathbf{x}, \mathbf{y}$ in the domain of T, and any scalar s,

$$T(\mathbf{x} + \mathbf{y}) = T(\mathbf{x}) + T(\mathbf{y})$$

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Is every line ($y = mx + b$) a linear transformation?

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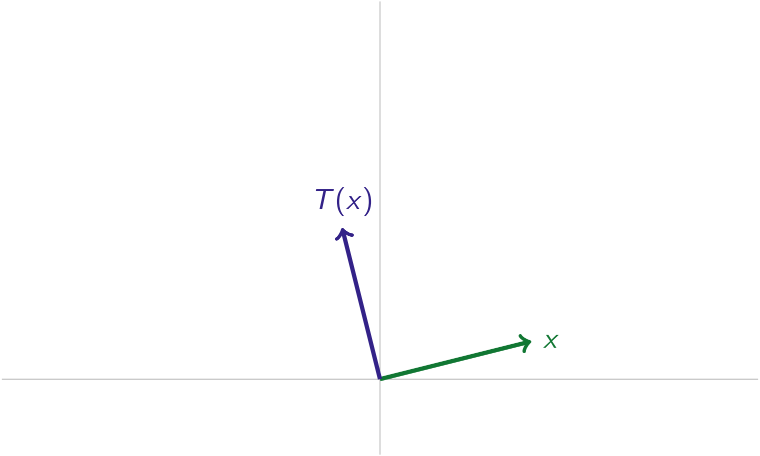

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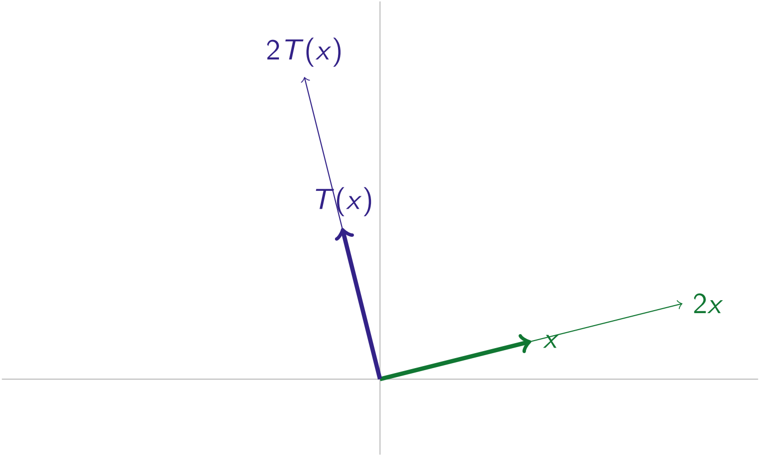

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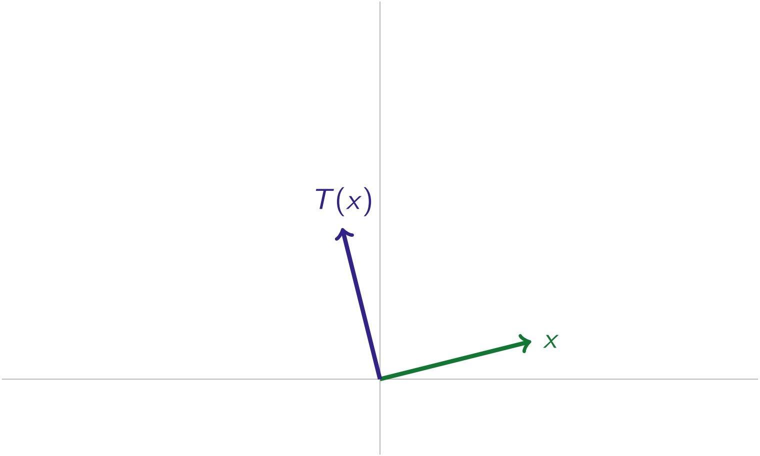

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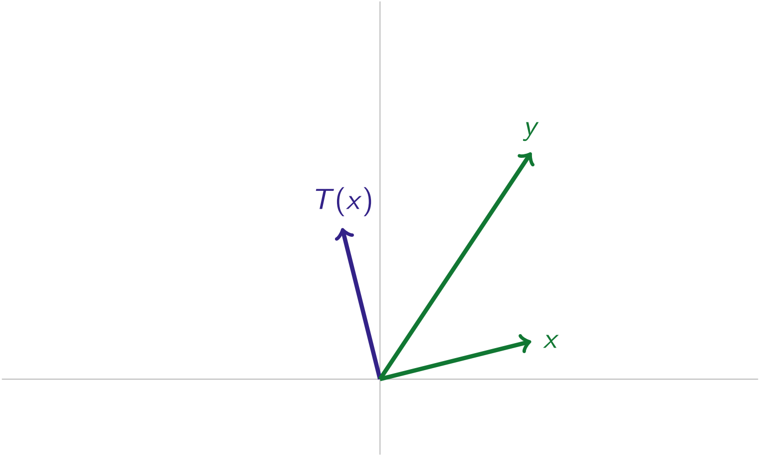

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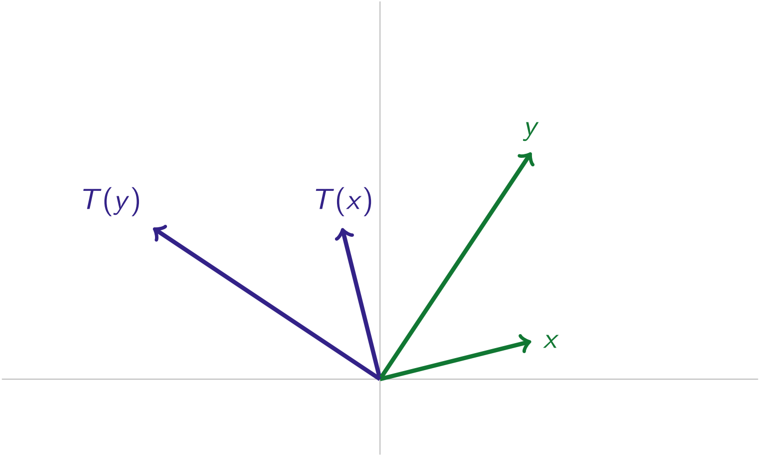

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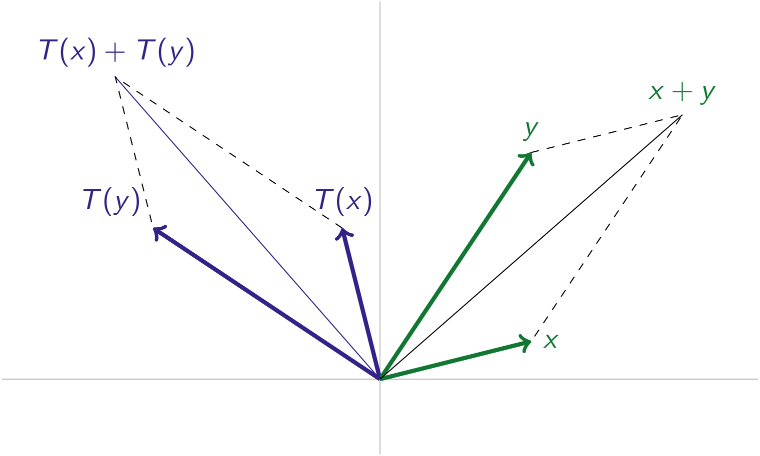

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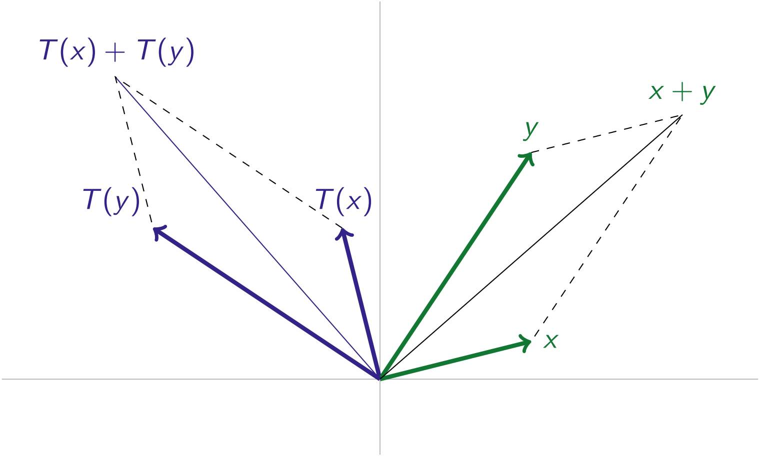
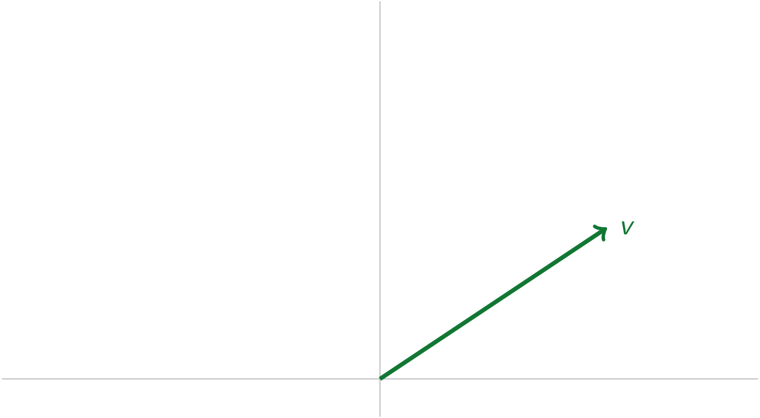


Rotation by a fixed angle is a linear transformation.

Computing Rotations



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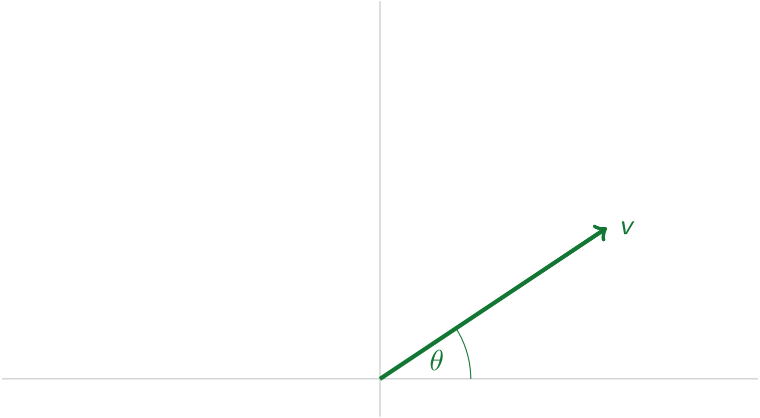

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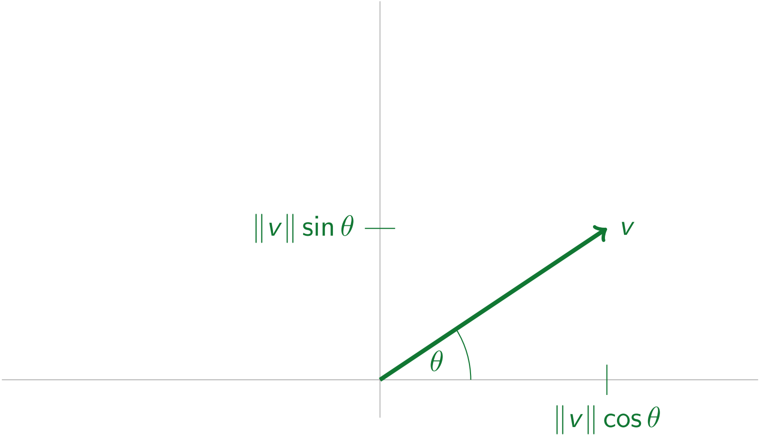

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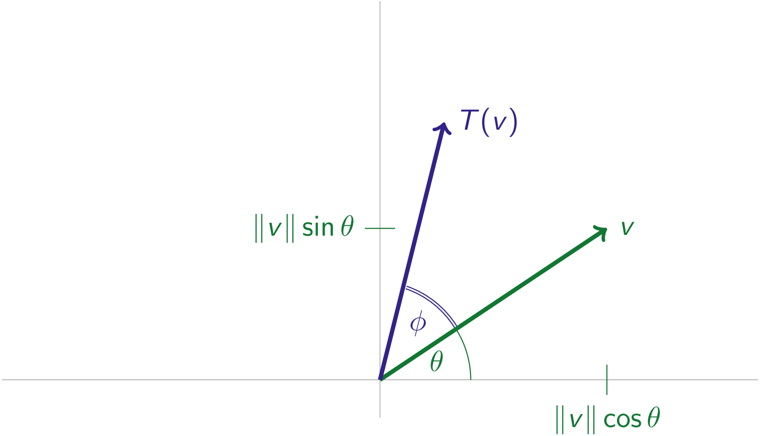

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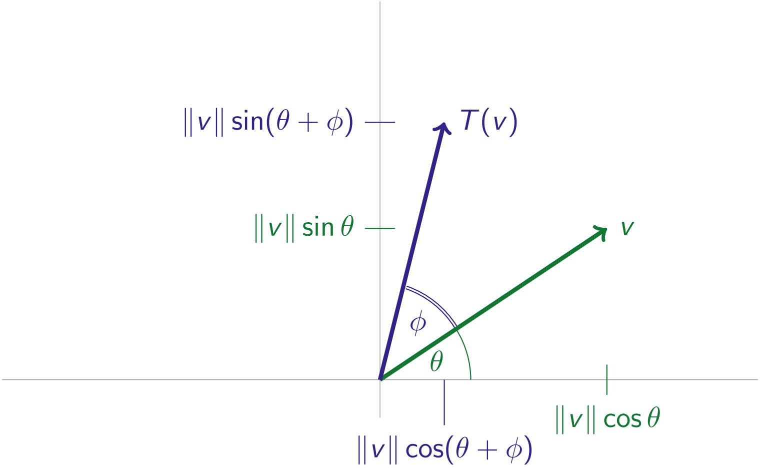

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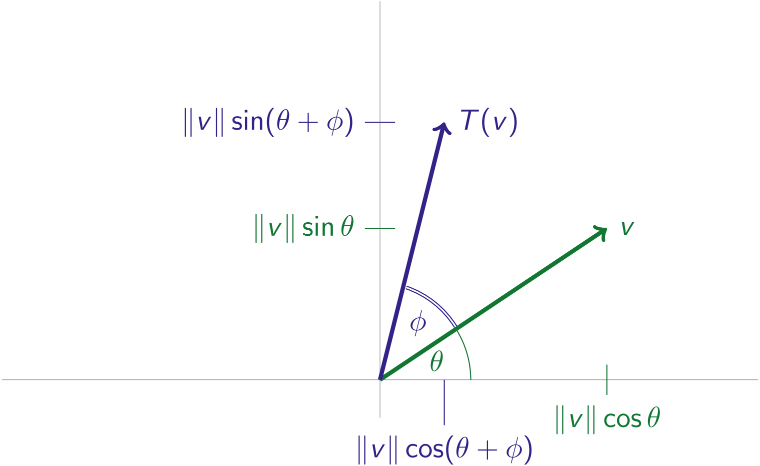
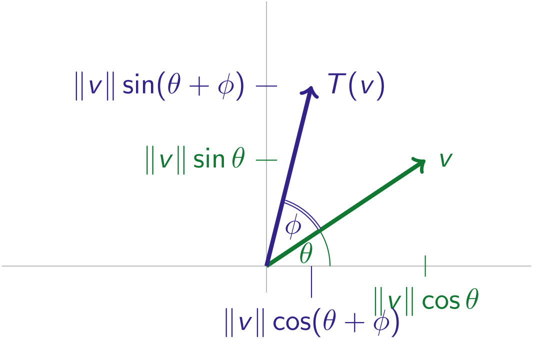


$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$ $\qquad$ $\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

Computing Rotations



$$v = [v_1, v_2]; \qquad T(v) = [x, y]$$

$$\begin{aligned} x &= \|v\| \cos(\theta + \phi) \\ &= \|v\| (\cos\theta \cos\phi - \sin\phi \sin\theta) \\ &= v_1 \cos\phi - v_2 \sin\phi \end{aligned}$$

$$\begin{aligned} y &= \|v\| \sin(\theta + \phi) \\ &= \|v\| (\sin\theta \cos\phi + \cos\theta \sin\phi) \\ &= v_1 \sin\phi + v_2 \cos\phi \end{aligned}$$

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$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

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$$\begin{aligned} x &= \|v\| \cos(\theta + \phi) \\ &= \|v\| (\cos\theta \cos\phi - \sin\phi \sin\theta) \\ &= v_1 \cos\phi - v_2 \sin\phi \end{aligned}$$

$$\begin{aligned} y &= \|v\| \sin(\theta + \phi) \\ &= \|v\| (\sin\theta \cos\phi + \cos\theta \sin\phi) \\ &= v_1 \sin\phi + v_2 \cos\phi \end{aligned}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

The matrix is called a rotation matrix, Rot_ϕ

Computing Rotations

$$\text{Rot}_\phi = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

What matrix should you multiply $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ by to rotate it 90 degrees?

Computing Rotations

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What matrix should you multiply $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ by to rotate it 30 degrees?

Computing Rotations

$$\mathrm{Rot}_\phi = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

What matrix should you multiply $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ by to rotate it 90 degrees?

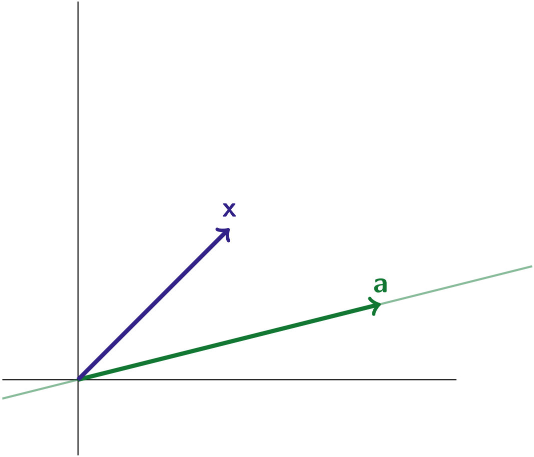
$$\mathrm{Rot}_{\pi/2} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

What matrix should you multiply $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ by to rotate it 30 degrees?

$$\mathrm{Rot}_{\pi/6} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

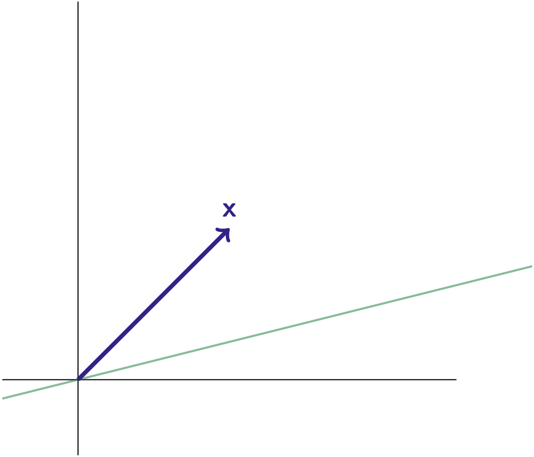
Projections

For a fixed vector $\mathbf{a}$, let $T(\mathbf{x}) = proj_{\mathbf{a}}\mathbf{x}$



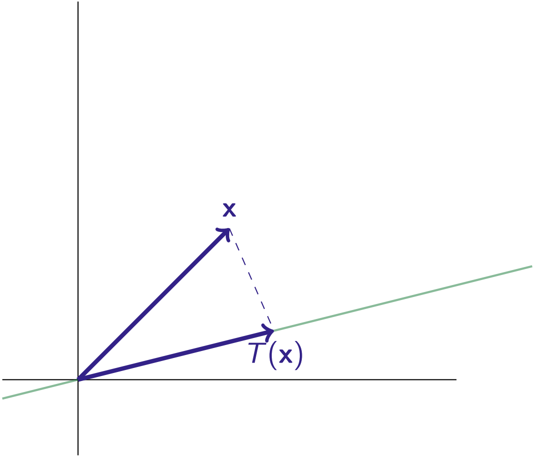
Projections

For a fixed vector **a** , let $T(\mathbf{x}) = proj_{\mathbf{a}}\mathbf{x}$



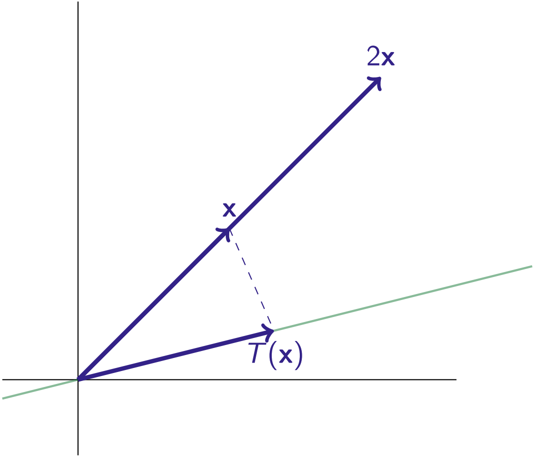
Projections

For a fixed vector **a** , let $T(\mathbf{x}) = proj_{\mathbf{a}}\mathbf{x}$



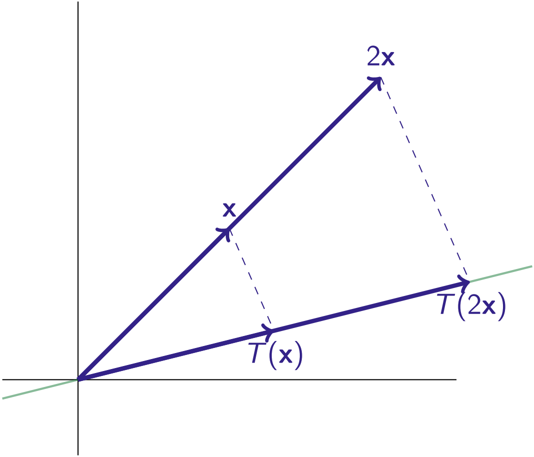
Projections

For a fixed vector $\mathbf{a}$, let $T(\mathbf{x}) = proj_{\mathbf{a}}\mathbf{x}$



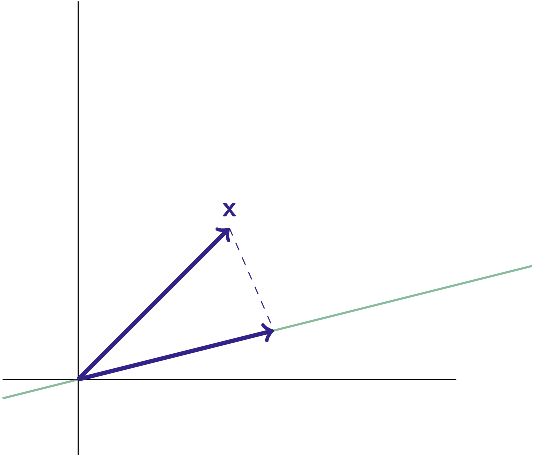
Projections

For a fixed vector **a** , let $T(\mathbf{x}) = proj_{\mathbf{a}}\mathbf{x}$



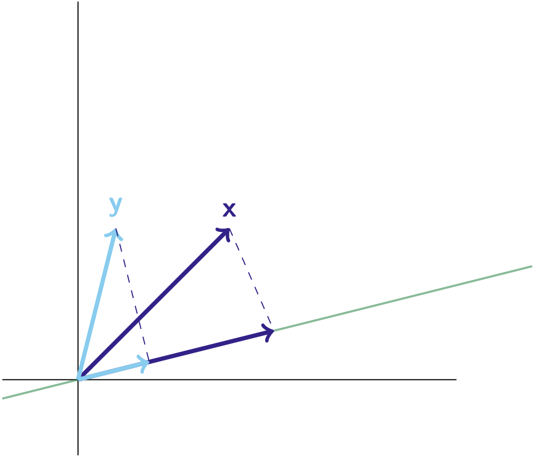
Projections

For a fixed vector **a** , let $T(\mathbf{x}) = proj_{\mathbf{a}}\mathbf{x}$



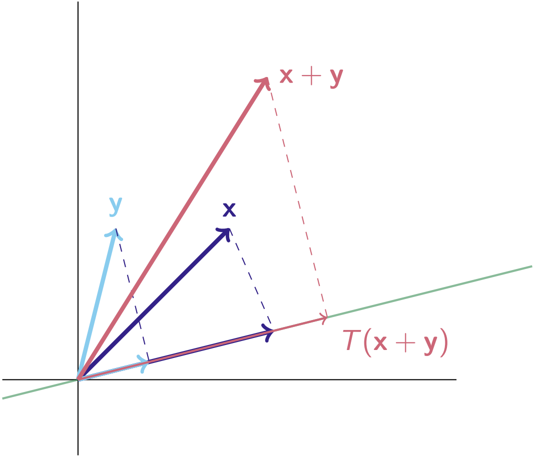
Projections

For a fixed vector $\mathbf{a}$, let $T(\mathbf{x}) = proj_{\mathbf{a}}\mathbf{x}$



Projections

For a fixed vector $\mathbf{a}$, let $T(\mathbf{x}) = proj_{\mathbf{a}}\mathbf{x}$



Computing Projections

Let $\mathbf{a} = [a_1, a_2]$ and $\mathbf{x} = [x_1, x_2]$.

$$proj_{\mathbf{a}}\mathbf{x} = \frac{1}{a_1^2 + a_2^2} \begin{bmatrix} a_1^2 & a_1 a_2 \\ a_1 a_2 & a_2^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$